



CHAPTER

3

AVERAGE

INTRODUCTION

“Average is a very simple but effective way of representing an entire group by a single value”.

$$\text{Average} = \frac{\text{Sum of all items in the group}}{\text{Number of items in the group}}$$

“Sum of all the items in the group” means “sum of the values of all the items in the group”.

A batsman’s performance can be expressed as the average number of runs scored per innings rather than giving the scores in individual innings. For example, let us say a cricketer scored the following runs in 9 different innings in a year: 35, 56, 124, 29, 0, 87, 98, 45 and 75.

Then his average score (per innings) for the years is

$$= \frac{35 + 56 + 124 + 29 + 0 + 87 + 98 + 45 + 75}{9} = 61$$

Similarly, if there are 60 students in a class, instead of talking of the height of each individual student, we can talk of “average” height of the class. The average height of the class of students is equal to the sum of the heights of all the students of the class divided by the number of students in the class.

Average is also called the “mean” or mean value of all the values.

POINTS TO REMEMBER

- (i) If the value of each item is increased by the same value p , then the average of the group or items will also increase by p .
- (ii) If the value of each item is decreased by the same value p , then the average of the group or items will also decrease by p .
- (iii) If the value of each item is multiplied by the same value p , then the average of the group or items will also get multiplied by p .
- (iv) If the value of each item is divided by the same value p ($p \neq 0$), then the average of the group or items will also get divided by p .
- (v) The average of a group of items will always lie between the smallest value in the group and largest value in the group – i.e., the average will be greater than the smallest value and less than the largest value in the group.

AN EASY METHOD TO CALCULATE AVERAGES

As already discussed, the average of a group of items whose values are given can be found out by the rule given at the beginning of this section. However, in most of the cases, we do not need to perform such elaborate additions and divisions. The calculation of averages can be simplified greatly by taking some arbitrary number (P) as a starting point, take the deviations (differences) of the given items (Q_i) from this arbitrary number, find the average of all these deviations ($Q_i - P$) and algebraically add it to the arbitrary number (P) to give the correct average of the given items.

If there are n items and they are denoted by $Q_1, Q_2, Q_3, \dots, Q_n$, then the average of these n items is given by

$$\text{Average} = P + \frac{1}{n} \sum_{i=1}^n (Q_i - P)$$

The extent to which this method will simplify the calculation will depend on the selection of the arbitrary value P . It should be selected in such a way that the positive and negative deviations cancel out each other to the extent possible. Then the final figure left for division will be relatively small making the division easier.

For example, the cricketer that we considered above scored the following runs in seven innings: 35, 56, 45, 43, 67, 70 and 48. Now, to find his average, we take an arbitrary figure, say 50 and first find the deviations of each of the scores from this figure. The deviations of the scores for 50 are -15, +6, -5, -7, +17, +20 and -2. The sum of these deviations is +14.

Hence the average of the cricketer's scores is = $50 + \frac{14}{7} = 52$

Please note that the number P (= 50 above) can be any value. Let us work out the same example taking a different value for P . Let us take P equal to 45. The deviations of the scores from P are -10, +11, 0, -2, +22, +25 and +3. The sum of these deviations is 49. Hence the average is $45 + \frac{49}{7} = 45 + 7 = 52$.

EXAMPLES

1. Find the average of the scores of the tests taken by Ram given that his scores were 78, 82, 84, 86, 87, 89, 93 and 95.

SOLUTION:

$$\text{Average} = \frac{\text{Total score in all the tests}}{\text{Number of tests taken}}$$

$$= \frac{78 + 82 + 84 + 86 + 87 + 89 + 93 + 95}{8} = 86.75$$

2. The monthly incomes of Raja, his wife and their son are Rs. 6000, Rs. 4000 and Rs. 1880 respectively. Find their average monthly income.

SOLUTION:

Average income

$$\frac{6000 + 4000 + 1880}{3} = \text{Rs.}3960$$

3. Rajiv purchased three dozen mangoes at Rs. 10 per dozen, two dozen mangoes at Rs. 15 per dozen and five dozen mangoes at Rs. 16 per dozen. Find the average cost per dozen of the mangoes that he purchased.

SOLUTION:

The cost of first three dozen mangoes = (3) (10) = Rs. 30 The cost of next two dozen mangoes = (2) (15) = Rs. 30

The cost of next five dozen mangoes = (5) (16) = Rs. 80 Total cost of the mangoes purchased = Rs. 140.

Average cost per dozen:

$$= \frac{\text{Total cost of Mangoes}}{\text{Number of dozen}} = \frac{140}{10} = \text{Rs.}14$$

4. The average age of 5 men is 20 years. Their average age increased by 1 year when a new man joined them. Find the age of the new man.

SOLUTION:

Total age of 5 men = (5) (20) = 100 years

Total age of 6 men = (6) (21) = 126 years The age of the new man = 126 – 100

i.e. 26 years

5. Six kilograms of wheat costing Rs. 18 per kg is mixed with nine kilograms of wheat costing Rs. 12 per kg. Find the price per kg of the mixture.

SOLUTION:

Total cost of 6 kg wheat = (6) (18) = Rs. 108

Total cost of 9 kg wheat = (9) (12) = Rs. 108 Average cost of the mixture

= Total cost / Total quantity

= 108 / 15 = Rs. 14.40 per kg.